

Separation and Reattachment in Transonic Airfoil Flow

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Results are presented of four research programs concerned with the phenomenon of shock boundary-layer interaction and ways that such interactions affect scaling of transonic airfoil flows. The results were obtained using a diverging nozzle and several simulated airfoil contours. Main variables of these programs were the freestream Mach number, the Reynolds number, the mode of boundary-layer transition, and the geometry and location of boundary-layer tripping devices. Emphasis in presenting the results is placed on the flow development that may lead to large transonic scale effects and on the dependency of this development on the unit Reynolds number and the initial boundary-layer condition. An attempt is made to relate components of the shock boundary-layer interaction, i.e., the pressure rise to shock-induced separation, the length of the separation bubble, and the extent of rear separation to parameters associated with the boundary-layer condition upstream of the shock.

Nomenclature

c	= airfoil chord
C_p	= pressure coefficient
$\langle C_p \rangle$	= $C_p(1 - M_\infty^2)^{1/2}$
C_{ps}	= pressure coefficient associated with separation
L_B	= length of shock-induced separated region (bubble)
M_∞	= freestream Mach number
M_1	= shock-upstream Mach number
M_L	= local Mach number
p	= static pressure
p_1	= shock-upstream pressure
p_s	= pressure at separation; pressure at the point where the shear layer leaves the surface (skin friction $\tau = 0$)
q	= dynamic pressure of undisturbed flow
Re	= unit Reynolds number
Re_δ	= Reynolds number based on boundary-layer thickness upstream of shock
Re_θ	= Reynolds number based on momentum thickness
t	= airfoil thickness
x	= distance measured from leading edge
$(x/c)_R$	= location of rear separation point
Z	= $1 - (x/c)_R$
δ	= boundary-layer thickness upstream of the shock
θ	= momentum thickness

Introduction

A NUMBER of investigations have recently established the possibility that transonic wind-tunnel data may yield misleading predictions of flight loads because of scaling effects. Such a problem is reported in Refs. 1 and 2. This scaling problem has arisen because modern transonic airfoils are different in several important respects from earlier designs. They tend to have relatively flat upper surface Mach number distributions over the forward portion of the airfoil at supercritical speeds, and they tend to be highly loaded near the trailing edge. The latter tendency carries with it the danger of trailing-edge separation or incipient separation. The former tendency means that small changes in pressure recovery over the aft portion of the airfoil can produce a substantial change in location of the shock wave terminating

the supersonic flow. The processes by which scaling effects may arise as a result of these tendencies are discussed in some detail in Ref. 3.

Although much has been learned about transonic-scaling phenomena since the experiences described in Ref. 1, aircraft designers do not yet possess the knowledge that permits the complete avoidance of such problems in the future. It is unlikely that they will possess such knowledge until better methods are obtained for viscous transonic air foil flow analysis. The research described herein has been done in an effort to provide a clear picture of the flow development leading to large transonic scale effects and to gather knowledge about flow details for which analytical methods do not yet exist. This research was designed to study details of shock-induced separation and reattachment phenomena on simulated airfoil contours. Particular attention was given to ways in which these phenomena are affected by initial conditions in terms of unit Reynolds number and boundary-layer properties.

The results discussed are drawn from four Lockheed-Georgia Company research programs dating from 1966. Two of these^{4,5} have already been reported in some detail. Data from these programs are included here where they augment more recent data and help to clarify general observations. One program⁶ has been reported only as an internal document. Most of the results discussed are drawn from the fourth program which has not heretofore been reported in the literature.

Flow Models

The flow phenomena contributing to transonic, scaling effects are depicted in Fig. 1. Basically the pattern comprises a strong and complex interaction between inviscid-transonic-airfoil flow and the boundary layer. It may be marked, as shown here, by shock-induced separation, a subsequent reattachment on the airfoil surface, and another boundary-layer separation near the trailing edge. In some cases the shock wave may not be strong enough to cause separation, but because the aft portion of the airfoil is highly loaded, separation may occur near the trailing edge. A third possibility is that only the separation bubble caused by the shock wave may be present. The ways in which these combinations may give rise to transonic scaling effects are discussed in some detail in Ref. 3. Summarizing briefly, significant scaling effects may arise when the separation patterns in low-Reynolds number wind-tunnel tests are different from those in flight.

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In the following sections, several features of the flow model are isolated and discussed, with attention focused on how these phenomena are affected by Reynolds number, boundary-layer properties upstream of the interaction, and the methods used in wind-tunnel tests to artificially trip and hence thicken the boundary layer.

Pressure Rise to Separation

In the early phases of this general work, attention was focused on the pressure rise to shock-induced separation to determine the role which this part of the over-all interaction might play in the scaling problem. Although we have concluded that this parameter is not very sensitive to Reynolds number, there is enough confusion in the literature about the magnitude and direction of the Reynolds number effect to warrant some discussion.

Data from Refs. 4 and 5 which were obtained with circular arc airfoils and in a diverging nozzle are plotted in Fig. 2 against the Reynolds number, based on initial boundary-layer thickness δ . The over-all trend of these data is to show practically no variation of separation pressure rise with Reynolds number. However, for the Ref. 4 data, there are small variations in Mach number which accompanied changes in Reynolds number, and it is difficult to sort out the slight effect of Reynolds number if one exists. The three points which were taken from the Mach number cross-plots of the Ref. 5 data do show a definite tendency for the separation pressure rise to decrease with increasing Reynolds number; it is difficult to ignore the decrease, even though it is slight.

Our purpose in pursuing this line further is that it is important for scaling of data to know at least the direction of the effect which an increase in Reynolds number will produce. To illustrate the confusion about this point which exists in the general literature, the reader is referred to Ref. 7 where, for supersonic flows over compression corners, the variation of the pressure rise to incipient separation with Reynolds number is opposite to the trend shown by Kuehn in Ref. 8. The authors of Ref. 7 state that there is no way of knowing, on the basis of existing data, whether the opposing trend for different ranges of Reynolds number is a real phenomenon or whether it arises from an experimental inadequacy.

In an effort to shed some light on this problem we have sought an analytical model which would adequately represent the significant features of shock-induced, turbulent, boundary-layer separation. In this search, the simplicity of Gadd's⁹ approach and the success with which this simple technique predicts the Mach number influence on separation pressure rise is intriguing. Gadd reasoned that, for the sudden deceleration of a turbulent boundary layer as it is exposed to a shock wave, only the thin portion of the boundary layer nearest the wall would be greatly influenced by viscous stresses, and that the outer portion of the boundary layer would behave as if it were an inviscid fluid. Stratford¹⁰ later developed a similar line of reasoning for the analysis of incompressible turbulent boundary layers in strong adverse pressure gra-

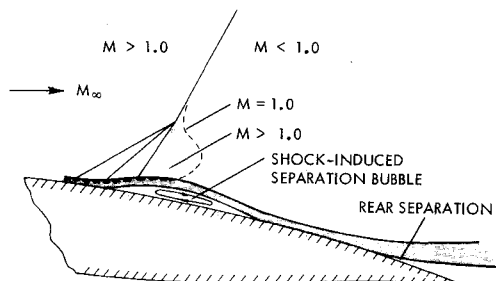


Fig. 1 Model of transonic shock boundary-layer interaction.

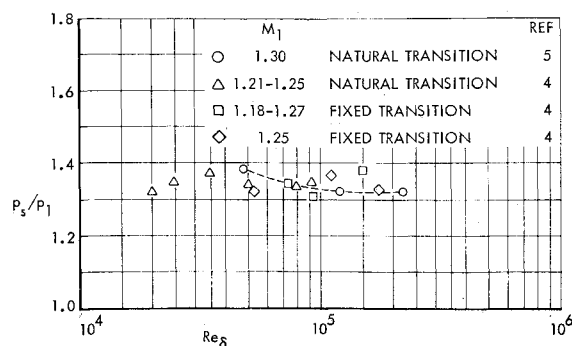


Fig. 2 Variation of separation pressure rise with Reynolds number.

dients, and Townsend¹¹ developed the mathematics of this model in some detail.

Halligan⁶ has modified the Townsend work to include consideration of the sublayer and has looked in some detail at the influence of Reynolds number on separation which this analysis yields. His work shows that the separation-pressure coefficient $C_{ps} = (P_s - P_1)/q$ is a function of Re_δ unit Reynolds number, and the pressure gradient at separation, or

$$C_{ps} = f[Re_\theta, Re/ft, dC_p/dx]$$

A plot from Halligan's report is shown as Fig. 3. In this plot, the effect of Reynolds number on the separation pressure coefficient is shown for several values of the nondimensional parameter $\theta(dC_p/dx)$. It is evident that the variation of C_{ps} with Reynolds number is not monotonic; with increasing Reynolds number, there is a decrease in C_{ps} , a minimum, and then an increase. Halligan goes on to show that C_{ps} may increase or decrease with increasing Re_δ , depending on how the increase in Re_δ is attained. If unit Reynolds number is held constant and δ is increased, C_{ps} will decrease with increasing Re_δ . On the other hand, if one holds plate length, for instance, constant and increases unit Reynolds number, C_{ps} will increase. Since the extension of this analysis to compressible flow has not yet been completed, no correlation with experimental data is attempted here. However, it does appear that this analytical model may yield results consistent with the trends of experimental data and may shed some further light on the Reynolds number effects problem.

Extent of Separated Regions

Figure 4 shows a sketch of the model and the model arrangement used to study separated regions associated with shock boundary-layer interaction. This simulated airfoil

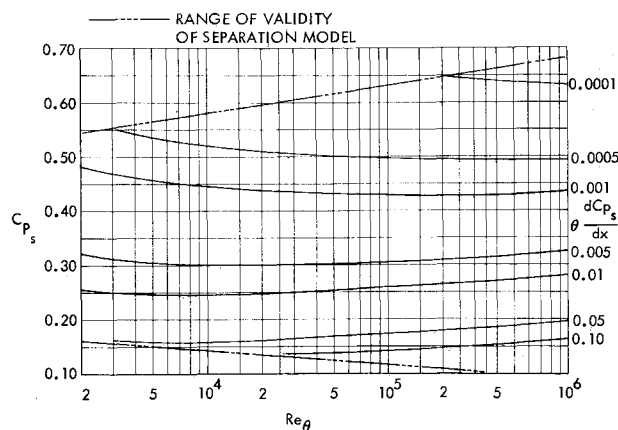


Fig. 3 Variation of separation pressure coefficient with Reynolds number (theory of Ref. 6).

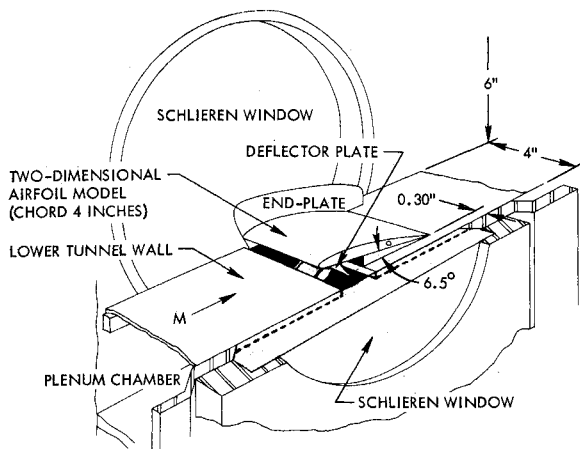


Fig. 4 Test set up for two-dimensional airfoil tests.

contour which was mounted on the lower test section wall of a small transonic wind tunnel¹² was developed by gradually changing the contour of the upper surface and the angle of attack until a configuration characterized by a relatively large sensitivity of the shock location to changes in the initial boundary-layer condition was found. The lower-wall boundary layer was removed along the lower surface of the model into a plenum chamber. The outflow of air from the latter was controlled so that the pressure coefficients upstream of the shock wave remained constant as the total pressure (unit Reynolds number) was changed. Gaps between the tunnel side-walls and the model in conjunction with end-plates were

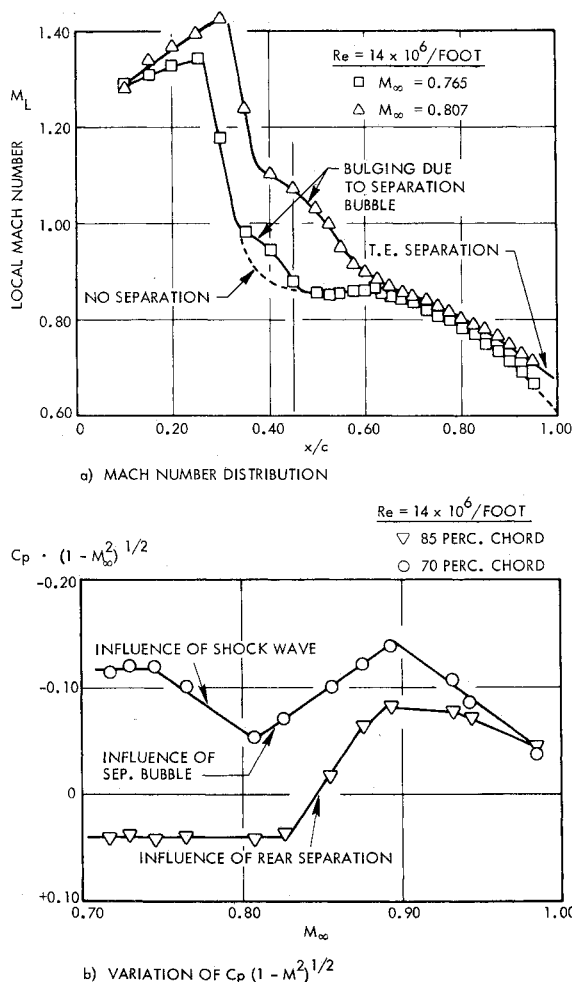


Fig. 5 Determination of separated regions.

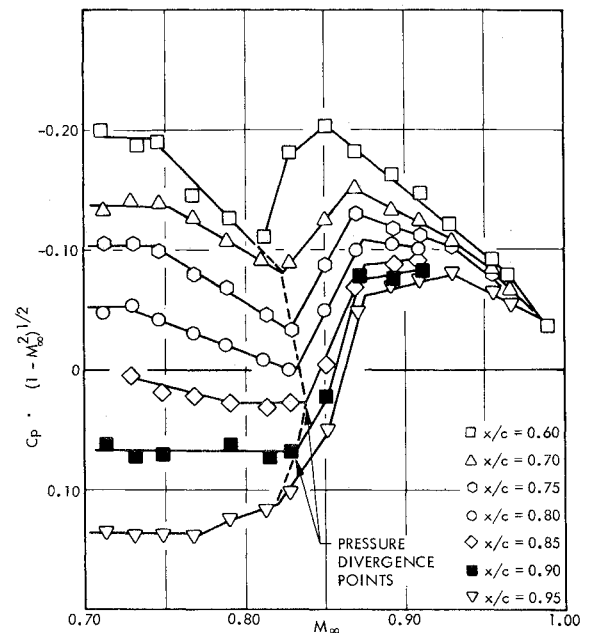


Fig. 6 Flow development at specific chord stations.

used to reduce three-dimensional effects resulting from the tunnel side-wall boundary layer. Sufficient two-dimensionality of the results was ensured by examination of oil flow pictures which showed a straight shock front over 75% of the span and only slightly curved (outboard) streamlines inside the separated regions. The test parameters of this investigation were Mach number, which was varied between 0.60 and 1.0; the unit Reynolds number, which ranged from 10×10^6 to 30×10^6 per ft; and free and fixed transition. Two tripping devices, both located at 5% chord, were used; a carborundum grit strip and triangular sandpaper patches, both spanning the entire model width. Data acquisition during the test program included pressure distributions and boundary-layer profile measurements at approximately 10 and 50% chord.

Determination of Separated Regions

Figure 5a shows chordwise Mach number distributions for freestream Mach numbers of 0.77 and 0.81. Flow separation at the foot of the shock is present at both Mach numbers, as indicated by the bulging of the curves downstream of the shock. The undisturbed pressure distribution is demonstrated by the dashed line. This characteristic behavior of the surface pressure (or Mach number) distribution led to the presentation of the data for a specific chord station in the form of $\langle Cp \rangle = Cp (1 - M_\infty^2)^{1/2}$ as function of the freestream Mach number (suggested in Ref. 3), as is demonstrated in Fig. 5b for the chordwise locations of 70 and 85%. It is evident that, for locations downstream of reattachment, $\langle Cp \rangle$ varies little with increasing Mach number until the effect of the approaching shock wave is felt by an increase in pressure coefficient. The subsequent drop indicates that the reattachment point moves across the chord location under consideration, here 70%. The Mach number independency of $\langle Cp \rangle$ at 85% chord followed by the rapid drop in pressure at higher freestream Mach numbers indicates that here rear separation moves upstream across the chord location considered.

The variation of $\langle Cp \rangle$ with freestream Mach number for chord locations ranging from 60–95% is shown in Fig. 6 for free transition at a unit Reynolds number of 20×10^6 . Following the dashed line connecting the separation and reattachment points (pressure divergence points) previously defined, it is evident that the separation bubble reaches 60% chord at a

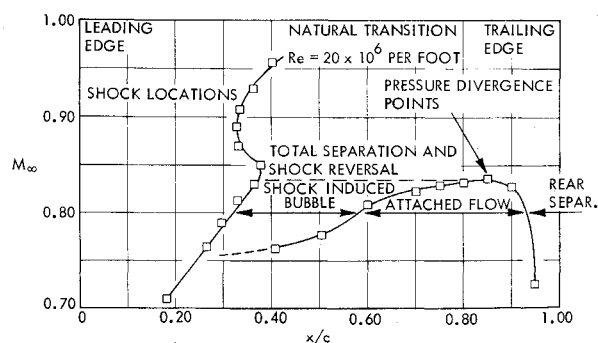


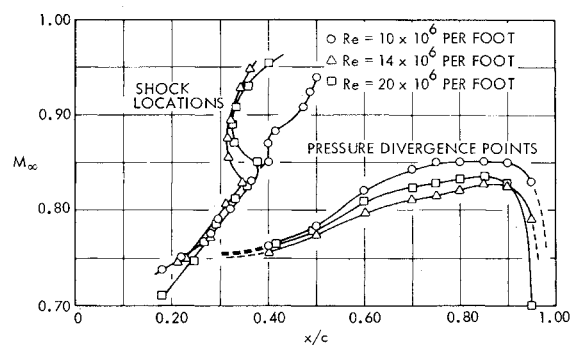
Fig. 7 Development of separated regions (definitions).

Mach number of approximately 0.807, whereas the rear separation point moves across 90% chord at about 0.83. The separated regions merge at a freestream Mach number of approximately 0.84. The gradual decrease in pressure preceding the rapid divergence in the case of the 95% chord station is caused by a gradual upstream movement of the rear separation point at Mach numbers between 0.77 and 0.82.

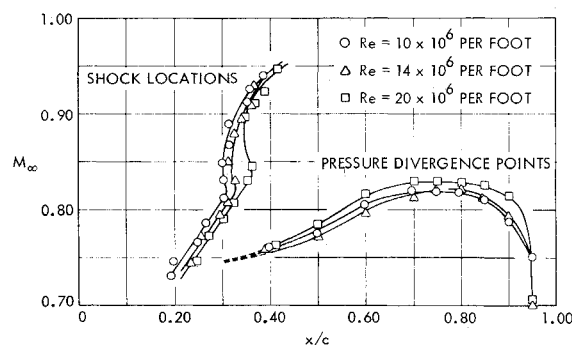
It seems reasonable to assume that, at a given freestream Mach number, the separation bubble extends from the foot of the shock to the pressure divergence point, while rear separation extends from the corresponding pressure divergence point to the wake. This led to the presentation of the chordwise location of the pressure divergence points together with the shock location as function of the freestream Mach number as shown in Fig. (7). Similar plots for all conditions investigated allow rapid determination of the extent of the separated regions and their development leading to the final "stall" of the flow on the airfoil. At any given Mach number, the bubble size is represented by the distance between the shock location and a corresponding point on the curve representing the pressure divergence points; attached flow exists over the chordwise range between corresponding points on the curve, and rear separation extends from the pressure divergence point to the trailing edge. Considering the flow development on an airfoil as represented by the curves in Fig. (7) one can see that a shock occurs on the airfoil and moves downstream as the freestream Mach number is increased. Rear separation develops, in the present example, before the shock becomes strong enough to cause the boundary layer to separate locally. A separation bubble appears at the foot of the shock and grows progressively with increasing Mach number; simultaneously, the rear separation point moves gradually upstream. At higher freestream Mach numbers, the growth rate of the separated regions increases, and total separation occurs as these regions merge. The shock-wave movement now slows down and subsequently reverses, a process which seems to be essential to the flow development leading to large scale effects. It should be noted here that, since the present model configuration does not allow interplay between the upper and the lower surfaces, changes in the trailing-edge pressure due to a local separation at the trailing edge do not seem to affect the shock location as would be the case in the presence of "circulation."

Effect of Unit Reynolds Number and Mode of Transition on the Development of Separation

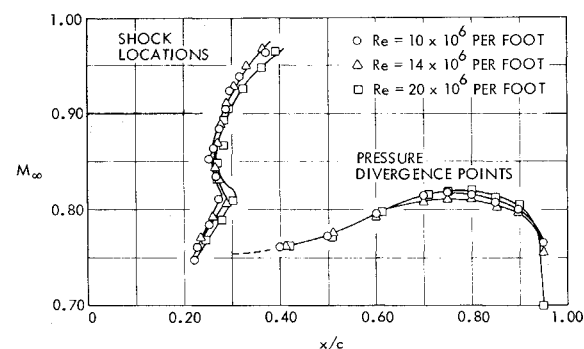
The variation of the shock location and the location of the pressure divergence points with freestream Mach number are shown in Fig. (8) for unit Reynolds numbers of 10, 14, and 20×10^6 and free and fixed transition. The development of the separated regions with increasing Mach number leading to total separation of the flow on the airfoil is essentially the same for the three modes of transition considered. Some amount of rear separation always occurs prior to the bubble development, which seems to be characteristic of either the



a.) NATURAL TRANSITION



b.) NO. 100 SANDPAPER WEDGES



c.) NO. 150 CARBORUNDUM GRIT

Fig. 8 Development of separated regions on a two-dimensional airfoil contour.

model contour or the test configuration. The effect of the unit Reynolds number is, in the Mach number range preceding total separation, restricted to small differences in the shock location and associated parameters, with variations similar to those observed on the circular-arc airfoils.⁴ The higher Reynolds number is, in this range, generally associated with a smaller bubble, due to a more rearward shock location as well as an upstream shift of the reattachment point. The flow development for natural transition at a unit Reynolds number of 10×10^6 is considered exceptional. Here, the shock assumes the most rearward position of all conditions investigated, and total separation occurs at a much higher freestream Mach number. The most likely explanation for the obviously optimistic results is that the boundary layer is laminar upstream of the shock and transition takes place within the interaction region. Favorable results due to transitional interaction are reported by Haines et al. in Ref. 13.

The variation in shock location and the location of the pressure divergence points with freestream Mach number are compared in Fig. (9) for free and fixed transition at Reynolds numbers of 20×10^6 and 10×10^6 , respectively. The development of the separation bubble is almost independent of the mode of transition, and the boundary-layer thickness which, in the case of free transition, was appreciably less than

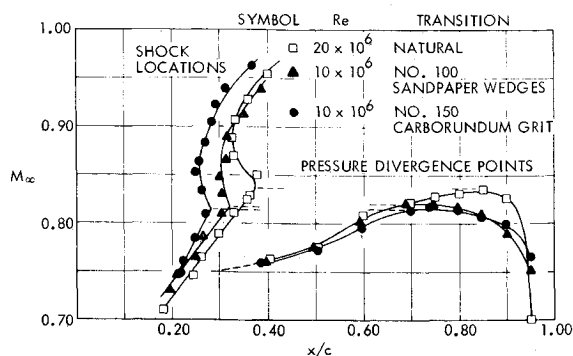
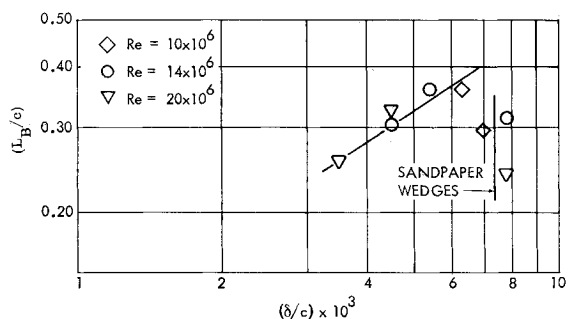


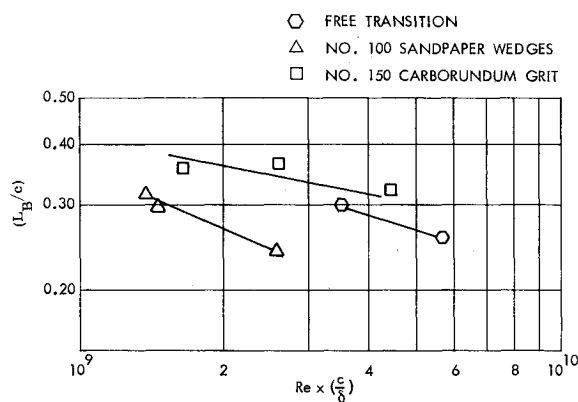
Fig. 9 Effect of fixing transition (boundary-layer thickness) on the development of the separated regions.

for fixed transition (free transition: $\delta \approx 0.016$ in; carborundum grit strips: $\delta \approx 0.023$ in; sandpaper patches: $\delta \approx 0.028$ in.).

The development of rear separation shows, contrary to the bubble, a fairly strong dependency on the initial boundary-layer condition and the spread of the rear separated region is, at the higher Mach numbers, much more rapid in the case of the thicker boundary layer. This results in the merger of the separated regions at a lower freestream Mach number and, consequently, in an earlier slowdown and reversal of the shock movement. These events seem to cause the large discrepancies in shock location observed on the present model and, similarly, on the C-141 wing.^{1,2} It can be seen that initial differences in shock location exist prior to the occurrence of total separation. The magnitude of these differences is similar to those observed in previous investigations, such as the one of Ref. 4 and measurements on configurations characterized by the presence of a shock-induced bubble only.¹³



a.) $(L_B/c) = f(\delta/c)$



b.) $(L_B/c) = f(Re \times \frac{\xi}{8})$

Fig. 10 Variation of the extent of the shock-induced separation bubble with the initial boundary-layer thickness and the unit Reynolds number.

Effect of the Reynolds Number and the Initial Boundary-Layer Condition on the Shock-Induced Separated Region

The preceding sections described the linking process between the shock-induced separation bubble and rear separation and showed, to some degree, the dependency of this process on the unit Reynolds number and the initial boundary-layer condition. In the following sections, an attempt is made to relate the extent of the bubble and the rear separated region to the unit Reynolds number and the boundary-layer thickness. It is realized that other parameters such as the airfoil shape, the freestream (or shock-upstream) Mach number, and the location of the shock (all three are, of course, related) are of equal importance; however, insufficient data are presently available to warrant a successful elimination of these parameters. It is, furthermore, realized that the results obtained with the sandpaper wedges have to be considered with caution since the wedges seem to act similar to vortex generators. In spite of these reservations, a correlation of the data with parameters associated with the boundary-layer condition upstream of the shock is presented.

Figure 10a shows the length of the separation bubble L_B/c as function of the boundary-layer thickness δ/c , for unit Reynolds numbers of 10, 14, and 20×10^6 and free and fixed transition. Close examination of the data indicates that a) in scaling, the boundary-layer thickness δ seems to be a much more important parameter than the unit Reynolds number (compare natural transition and carborundum grit tripping device), b) the use of devices which create a considerable increase in turbulent mixing (like wedges, vortex generators, etc.) introduces an unknown which we do not know how to scale, and c) for any one given device, an increase in unit Reynolds number seems to result in a decrease in the extent of the separated region.

The opposite effect of the unit Reynolds number and the boundary-layer thickness led to the formulation of the parameter $Re \times c/\delta$ Fig. 10b. This parameter seems to correlate bubble sizes quite well and seems to point out changes resulting from the different characteristics of the tripping devices. These differences are, of course, one reason why one cannot expect a duplication of the parameter $Re \times c/\delta$ in the wind tunnel to automatically give results representative of full-scale condition; however, the data of Figs. 10, 12, and 13, the later showing the variation of rear separation with $Re \times c/\delta$, point out the direction in which one has to proceed in order to obtain full-scale results in the wind tunnel.

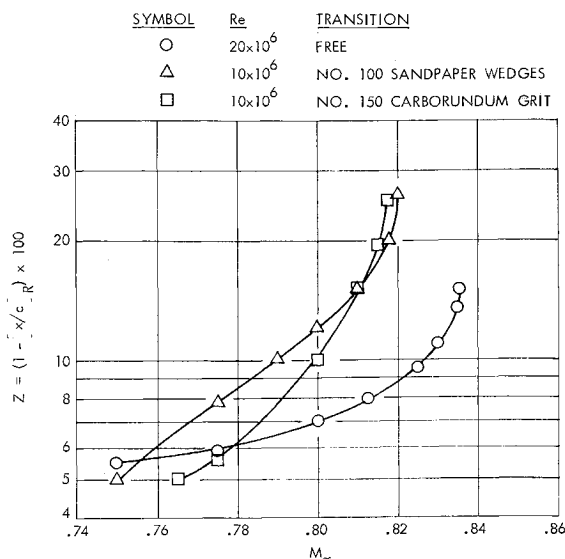


Fig. 11 Development of rear separation with increasing Mach number.

Effect of the Unit Reynolds Number and the Initial Boundary-Layer Condition on Rear Separation

It was demonstrated that the flow development leading to large scale effects is dominated by rear separation and its sensitivity to changes in the initial boundary-layer condition. We want to re-emphasize here that the results presented were not obtained on a real airfoil but on the model configuration of Fig. 4. It is, therefore, quite possible that rear separation occurred initially due to this particular model arrangement. Furthermore, one should not attempt to obtain general quantitative dimensions from these data. However, the trends observed are believed to be correct and generally valid. The nondimensional distance of the rear separation point from the trailing edge as function of the freestream Mach number is plotted in Fig. 11 for free and fixed transition at unit Reynolds numbers of 20×10^6 and 10×10^6 , respectively. It can be seen that the crucial difference in the development of the separated regions is the rapid increase in growth rate in the case of fixed transition at the higher freestream Mach numbers.

Figure 12 summarizes the effect of the unit Reynolds number and the boundary-layer condition on rear separation. The sensitivity of rear separation dZ/dM is plotted here as function of $Re \, xc/\delta$ for all test conditions of the present investigation at a freestream Mach number of 0.80. It can be seen that the sensitivity decreases with increasing $Re \, xc/\delta$. A comparison of the shock locations in Fig. 9 at freestream conditions for which total separation has occurred shows that the largest deviations in shock location are associated with the configuration showing the highest sensitivity of rear separation.

Figure 13 compares the variation of Z , representing the extent of the rear separated region, with $Re \, xc/\delta$ for the present air-foil contour with analytical results obtained by F. Thomas¹⁴ for a two-dimensional airfoil (RAE 103). Both sets of data show the same Reynolds number dependency: a decrease in the extent of rear separation with increasing $Re \, xc/\delta$. It should be noted that the scatter of the present data is believed to be mainly due to difficulties in determining the rear separation point.

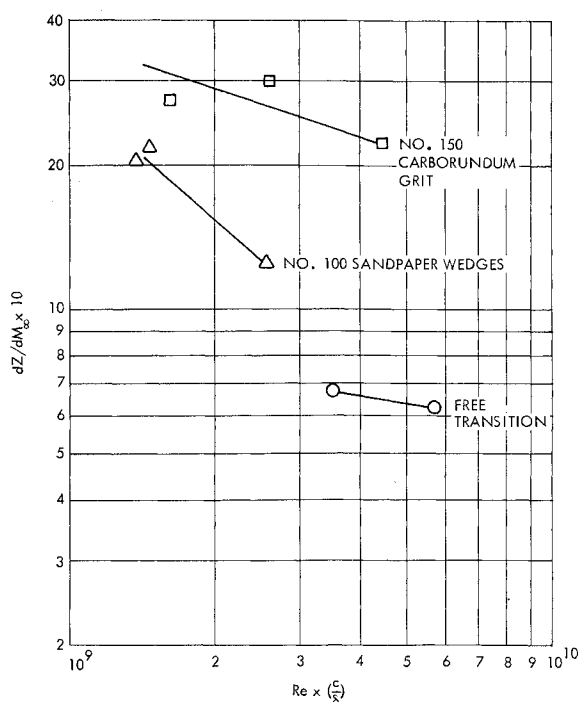


Fig. 12 Sensitivity of rear separation to changes in the initial boundary-layer condition ($M_\infty = 0.80$).

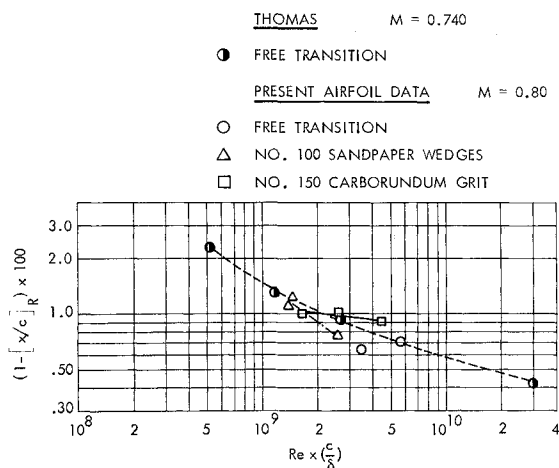


Fig. 13 Variation of the extent of rear separation with $Re \, xc/\delta$.

Again, duplicating $Re \, xc/\delta$ in the wind tunnel will not automatically give results representative of full-scale conditions. Other parameters, as was indicated in the preceding section, have a strong influence on the extent and the development of the rear separated region. This becomes particularly obvious by considering the sensitivity, dZ/dM in Fig. 12 where appreciable differences exist in the magnitude of dZ/dM between natural transition and the carborundum grit tripping device at the same value of $Re \, xc/\delta$; however, in both cases a decrease in dZ/dM with increasing $Re \, xc/\delta$ is indicated.

Thomas developed his analytical method to predict the onset of buffeting on a given airfoil. His approach is based on Sinnott's semiempirical method for the prediction of the transonic pressure distribution¹⁵ in conjunction with the boundary-layer theory advanced by A. Walz.¹⁶ The computation is carried through the shock boundary-layer interaction region by spreading the pressure rise across the shock over several boundary-layer thicknesses, under the assumption that the height of the separation bubble is small compared to the thickness of the shear layer. Furthermore, Thomas assumes that rear separation is predominant, spreading upstream to the foot of the shock with increasing Mach number. Thomas' results agree quite well with wind tunnel as well as flight data, indicating that his approach properly accounts for Reynolds number effects. It is unfortunate that the method, in its present form, cannot be applied to cases where a strong interplay exists between rear separation and the shock-induced bubble. An extension of the present method to better account for changes in the boundary-layer parameters due to the interaction with the shock wave, and the introduction of more advanced methods to determine the transonic-flow development are extremely desirable. Such a method would, as is already pointed out in Ref. 3, be an invaluable tool for the assessment of scale effects by determining the occurrence and extent of rear separation under full-scale conditions.

Conclusions

This report presents results of research programs conducted to investigate the over-all flow development leading to significant scale effects in the transonic speed range and to study, in general, parameters associated with separated flows and their dependency on the initial boundary-layer condition.

The results have, in support of the postulated flow model of Ref. 3, indicated that large scale effects can occur due to the development of rear separation and its interaction with the shock-induced separation bubble. The key factor here was found to be the large sensitivity of the development of rear separation to changes in the initial boundary-layer condition

and, in particular, to changes in the boundary-layer thickness. The thicker boundary layer was found to cause a more rapid upstream spread of the rear separation and an earlier (i.e., at a lower freestream Mach number) slowdown and reversal of the shock movement. The contribution of the bubble development, other than modifying the boundary-layer characteristics, is not immediately obvious from the present results. However, it appears that, in the absence of severe rear separation, the bubble development in the case of a turbulent boundary-layer upstream of the shock does not result in any major scale effects. This is mainly due to the rapid growth of the bubble with increasing Mach number immediately preceding total separation on the airfoil.

The pressure rise to separation at the shock appears even less sensitive to scaling effects in the transonic Mach number range than at higher Mach numbers. There is still confusion, however, about the direction of the effect. A preliminary analysis based on an equilibrium layer approach indicates that this confusion may stem from the opposite effects of increasing boundary-layer thickness and increasing unit Reynolds number.

On the basis of the limited data available, the length of the shock-induced separated region on the airfoil was found to decrease with increasing unit Reynolds number, whereas increasing the boundary-layer thickness resulted in an increase in the extent of the bubble. These trends are believed to be correct. However, since airfoil shape will certainly be important, no attempt to obtain general quantitative dimensions from these data should be made.

The rear separation seems to behave essentially as the bubble; an increase in unit Reynolds number resulted in a downstream movement of the rear separation point, while an increase in boundary-layer thickness had the opposite effect. The trend shown by the present data is in agreement with analytical results obtained by F. Thomas.¹⁴

It is believed that the amount of data presently available is insufficient to form strong conclusions about many of the aforementioned trends. Further experimental and analytical results covering a wide range of Reynolds numbers and geometric configurations, including airfoils and wings as well as wedge-type interactions, are required to define the behavior of separated flow in general and to assess the occurrence and extent of scale effects at transonic speeds.

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